

Electromagnetic Gating in Ion Channels

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There have been many attempts to develop a theoretical explanation of the phenomena of electromagnetic field interactions with biological systems. None of the reported efforts have been entirely successful in accounting for the observed experimental results, in particular with respect to the reports of interactions between extremely low frequency (ELF) magnetic fields and biological systems at ion cyclotron resonance frequencies. The approach used in this paper starts with the Lorentz force equation, but use is made of cylindrical co-ordinates and cylindrical boundary conditions in an attempt to more closely model the walls of an ion channel. The equations of motion of an ion that result from this approach suggest that the inside shape of the channel plus the ELF magnetic fields at specific frequencies and amplitudes could act as a gate to control the movement of the ion across the cell membrane.

Introduction

The Lorentz force equation has been the starting point for a number of attempts to analyze electromagnetic field interactions with ions in biosystems (Chiabrera *et al.*, 1985; Liboff, 1985; McLeod & Liboff, 1986; Durney *et al.*, 1988; Liboff & McLeod, 1988*a*). For the most part, these approaches have been overly mechanistic, not involving the more usual biophysical considerations normally used in describing ion transport (e.g. counterion electrostatics, thermodynamic constraints and molecular dynamics). Nevertheless, these approaches have yielded models consistent with various reports (Liboff *et al.*, 1987; McLeod *et al.*, 1987; Pilla *et al.*, 1987; Smith *et al.*, 1988*a, b*; Thomas *et al.*, 1988; Lyle *et al.*, 1991; Reese, *et al.*, 1991) indicating the existence of low frequency (ELF) magnetic field interactions with biosystems at ion cyclotron resonance frequencies, i.e. at frequencies corresponding to charge to mass ratios of ions such as Ca^{2+} , Mg^{2+} and K^{+} . A recent approach by V. V. Lednev (Lednev, 1991) offers a model that focuses on Ca-binding proteins, suggesting the existence of coherent, magnetically-induced splitting of vibrational levels, with the probability of transition frequencies occurring at cyclotron resonance.

By contrast, this group has argued that the most likely site for such ELF interactions is the ion channel (McLeod & Liboff, 1986, 1987*a*; Liboff & McLeod, 1988*a*). Membrane channels involve well-defined current paths for which collision kinetics

are less critical than in the cytosol or extracellular environment. Further, they entail periodic structures that are consistent with the low-frequency resonance behavior observed in the magnetic interactions. Regardless of the details of the resonance interaction, whether involving proteins that are either intrinsically membrane-bound, or removed from the membrane, there are theoretical difficulties in conceiving of a coupling process dependent on energies far less than the thermal kT level.

Major experimental data that remain unexplained by most theoretical efforts to date include first, a pattern of responses by a biosystem to specific harmonics (McLeod & Liboff, 1987b; McLeod *et al.*, 1987; Liboff *et al.*, 1990), (e.g. one, three, five and 15 times the calculated fundamental frequency), and second, the functional dependence of the overall response on the amplitude of the applied AC field (Liboff *et al.*, 1987, 1990; Smith *et al.*, 1988b). Other data that must be explained by a successful theory include resonant frequency curves, specific ion coupling, and a biosystem resonance response at any frequency set by choosing any d.c. magnetic field intensity up to about 50 microTesla (μT) (McLeod *et al.*, 1987).

In the present paper, we propose a radically different approach to the question of how ion gating may occur in channels. We suggest that the efficacy of the ELF interaction may have less to do with energy accretion (Liboff & McLeod, 1988a) and more to do with providing selective pathways through the channel, such that the successful transport of an ion is dependent on the proper choice of field parameters. This becomes evident when we explore solutions to the Lorentz force equation as written in cylindrical co-ordinates, making use of symmetries that are presumably connected to the ion channel lumen (Liboff & McLeod, 1988b, c). Unlike our previous attempts, we do not assume an initial sinusoidal time variation for the ion in question, but instead consider an ion that is delivered to the mouth of the channel with zero kinetic energy and which is subsequently accelerated by the transmembrane potential difference or the chemical gradient. This new approach leads to time-dependent equations that are separable in the radius and angle variables (r, ϕ) plus a third equation for z , taken to be along the channel axis. We then apply boundary and initial conditions on the ion as dictated by a simplistic channel model with cylindrical symmetry. It is subsequently found that the path of the center of mass of the ion is helical along the z axis, but that in addition the *radius* of this path varies with z . The period and amplitude of this variation in the radial direction depend on the magnitudes of both the d.c. and a.c. magnetic field as well as on the a.c. frequency.

The resulting solutions are highly suggestive. In particular, if the variations in the radial direction amplitude of the helix are comparable to the intrinsic cross-sectional shape of this simple channel (e.g. the constraining walls), then the permeability of the ion becomes sharply dependent on a narrowly defined selection of magnetic field parameters. This also leads to (a) specific allowed harmonics of the fundamental resonance frequency, (b) specific optimum a.c. magnetic field intensities, and (c) an upper limit on the magnitude of the d.c. magnetic field.

The cross-sectional shape that is postulated for the channel lumen follows that recently reported for the acetylcholine receptor channel (Toyoshima & Unwin, 1988) (Fig. 1). A similar shape has been deduced for the potassium channel of the sarcoplasmic reticulum (Latorre & Miller, 1983; Miller, 1983; Miller *et al.*, 1984).

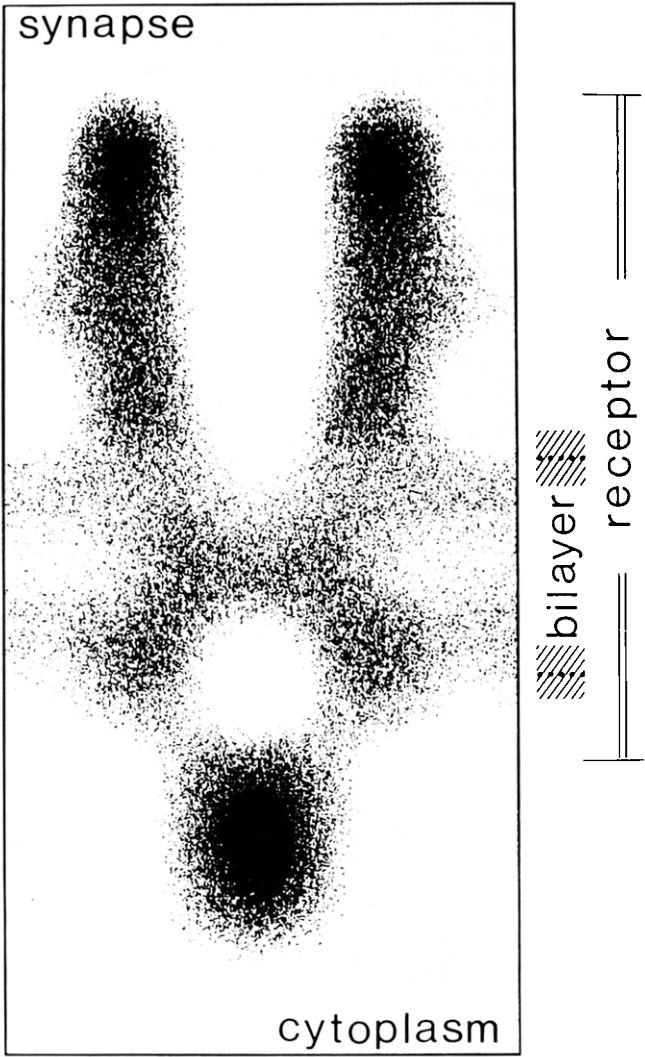


FIG. 1. A cross section of the acetylcholine receptor channel obtained from electron diffraction reconstruction techniques. Used with kind permission of N. Unwin.

Theory

Most of the details of the analysis appear in Appendix A. The starting point is taken to be an ion initially at rest at the center of the mouth of a cylindrically symmetric channel. The motivation for the analysis is to try and understand how the ion might move through the channel after leaving the binding site when limits on ionic movement such as the constraining channel walls, the presence of applied magnetic fields and certain internal restoring forces are included in the calculations. A very simplistic model of the channel is developed by using the mathematics and our diatom data (McLeod *et al.*, 1987; Smith *et al.*, 1988b). The results obtained suggest that for a specific set of assumed conditions, a novel type of ion "gating" can occur that would appear from external observations to closely follow a classical resonance curve (if one looks at biosystem response versus applied frequency for instance) for a particular ion. This "gating" is quite sensitive to the internal shape of the channel. The results calculated using this model appear to fit virtually all the experimental data obtained by our group. In addition, the activity or inactivity of these unique channels as a cell attempts to return to equilibrium could also suggest why a particular biological system responds (or does not respond) to ELF fields if the presence of the channels can be shown to be a function of some perturbation applied to the system (Blackman *et al.*, 1991; McLeod *et al.*, 1992).

From the Lorentz force equation written in cylindrical co-ordinates, the movement of the center of mass of the ion is found to be described by

$$r = \frac{(a - r_0)}{A} \exp(-t/\tau) \left[\sinh\left(\frac{At}{2\tau}\right) + A \cosh\left(\frac{At}{2\tau}\right) \right] + (r_0 - a) \quad (1)$$

and

$$\phi \approx -\frac{\omega_{B0}}{2} t = \frac{qB_0}{2m} t \quad (2)$$

and

$$z \approx \left[\tau(q/m) \left(E_z - \frac{F_z}{q} \right) \right] t \quad (3)$$

where

$$A = \sqrt{1 - 8\tau^2\omega_B^2} \quad (4)$$

All the terms are defined in Appendix A. Note the movement in the z direction is of the form

$$z = (V)t \quad (5)$$

with (V) being the term in the square brackets in eqn (3).

It is seen that these equations denote that the ion moves down the channel (in the z direction) in a helical path, but the radius [eqn (1)] of the helix is changing in time (i.e. as the ion moves through the channel). To study what eqn (1) suggests about

the ion movement, the z directed velocity was initially assumed to be approximately

$$V = \frac{\text{the channel length}}{\text{the time for one cycle at 16 hertz}} \quad (6)$$

The frequency of 16 Hz was chosen based on our experimental results (McLeod *et al.*, 1987; Smith *et al.*, 1988b). Initial plots of eqn (1) from zero to 100 msec (i.e. over at least one cycle in the 10–100 Hz range) indicated that the locus of points for the radius of the helical path of movement of the ion had a distinctive characteristic shape that was sensitive to both the a.c. and d.c. magnetic field amplitudes, as well as the ionic charge to mass ratio and the applied frequency. Further study indicated that if the velocity in the z direction had a specific value, and if the channel interior was shaped such that two narrow obstructions were present at just the right z positions, a form of ion resonance would be observed. The shape of the inside of the channel thus obtained was similar to that reported by Toyoshima & Unwin (1988) and shown in Fig. 1. The depth and axial position of the obstruction inside our channel model was adjusted to obtain the best fit to the available experimental data with the position obtained being about 62% from the luminal face, and having a depth equal to about 80% of the channel diameter (we later found that Miller and his colleagues (Latorre & Miller, 1983; Miller, 1983; Miller *et al.*, 1984) had postulated such a restriction at 65% of the length in the potassium channel of the sarcoplasmic reticulum).

Figures 2–5 illustrate the ion movement in the radial and axial directions as the applied magnetic field parameters are varied. Experimental data had indicated that resonance type behavior could be achieved for any d.c. magnetic field up to about 50 μT in one of our laboratory biosystems (the *Amphora coffeaeformis*). This feature could also be obtained in the above model if the (τ) term in eqns (1–4) was assumed to have the form

$$\tau = (C)B \quad (7)$$

where

$$C = \text{a constant}$$

and

$$B = \text{the d.c. magnetic field intensity.}$$

Discussion

The solution developed above for the ionic motion is by no means unique, mainly because of the several simplifying assumptions that are employed to obtain a closed solution for our original differential equations. Starting the ion from rest at the mouth of the channel is different from previous attempts along these lines, and could represent a more realistic picture of the initial energetics of the ions. That is, the most economical way to deliver the ion to the mouth of the channel is for the ion to have, at the onset, the least kinetic energy. The acceleration of the ion in the z

direction results directly from the potential difference across the membrane. Although we again find helical motion through the channel, this helicity does not result from the intrinsic protein structure. Instead, the protein structure can act as a filter for different helical paths. Not only is the pitch of the protein structure a constraint on the motion as we have previously argued (Liboff & McLeod, 1988a), but the overall shape of the channel lumen is also important. Thus, we find that the radius of the ion's helical path can be sharply modulated with a weak a.c. magnetic field having certain frequencies at a given amplitude. In effect, the shape of the channel lumen provides the feature that limits the frequency response bandwidth of the biosystem to a rather narrow window. This "narrowbanding" feature could provide a way to overcome the kT problem in a manner similar to that suggested by Weaver & Astumian (1990) for electric field interactions.

The motion of the center of mass of the ion is illustrated in Figs 2–5. In each case, the simplistic channel is shown with the cross-hatched walls representing protein, and the symmetry axis being along the centerline of the hourglass shaped channel. Each channel is longitudinally sliced in half with the outside of the membrane on the left and the cell interior on the right. The drawings are scaled so that the channel length is about 50 Å and the diameter of the channel at the input (left side) is about 10 Å. The overall shape of the channel is very similar to the cross-section determined by Toyoshima & Unwin (1988) for the nicotinic acetylcholine receptor channel (Fig. 1). The symmetrical pair of lines that deviate from the central axis in each sketch represent the cross-sections of the helical paths as calculated from eqn (1). Each picture thus shows the envelope of the helical progress of the ion along the channel. In each figure tau (τ) is set according to the form assumed in eqn (7). The velocity of the ion in the z direction is set as discussed above, which results in about 8.7×10^{-8} m sec $^{-1}$. Each figure therefore plots the path of the ion under conditions of cylindrical symmetry when the time through the channel corresponds to about one complete cycle at resonance.

The paths that are shown illustrate the key point that the radius of each helical path is modulated (in the z direction) differently by different applied magnetic fields. Only for certain appropriate fields will the ion pass through the channel in an "unhindered" manner. It is apparent that there is a "lock-and-key" feature embodied in this analysis relating the cross-sectional shape of the channel lumen to the ionic path. In effect, this lays the foundation for a new type of gate mechanism for controlling the transport of ions through membrane channels. It is different from previous mechanisms in that it is not dependent on binding sites, conformational changes, osmolality, or other *biochemical* factors, but instead involves *electromagnetic* field parameters including the ionic charge to mass ratio. It is appropriate to call this *electromagnetic gating*. Note that the ionic path fits the channel at resonance, but that this fit becomes less and less perfect as one moves away from (on either side of) resonance. The sharpness of the resonance, (i.e. the change in conductance) is governed by the width of the channel at the input and the output, as well as the width in the constricted region. It is interesting to note that if this constriction is very narrow, then our computer-drawn paths indicate a "fit" at harmonics 1, 3, 5 and 15 for the above conditions, and a less than perfect fit for all even harmonics as well as

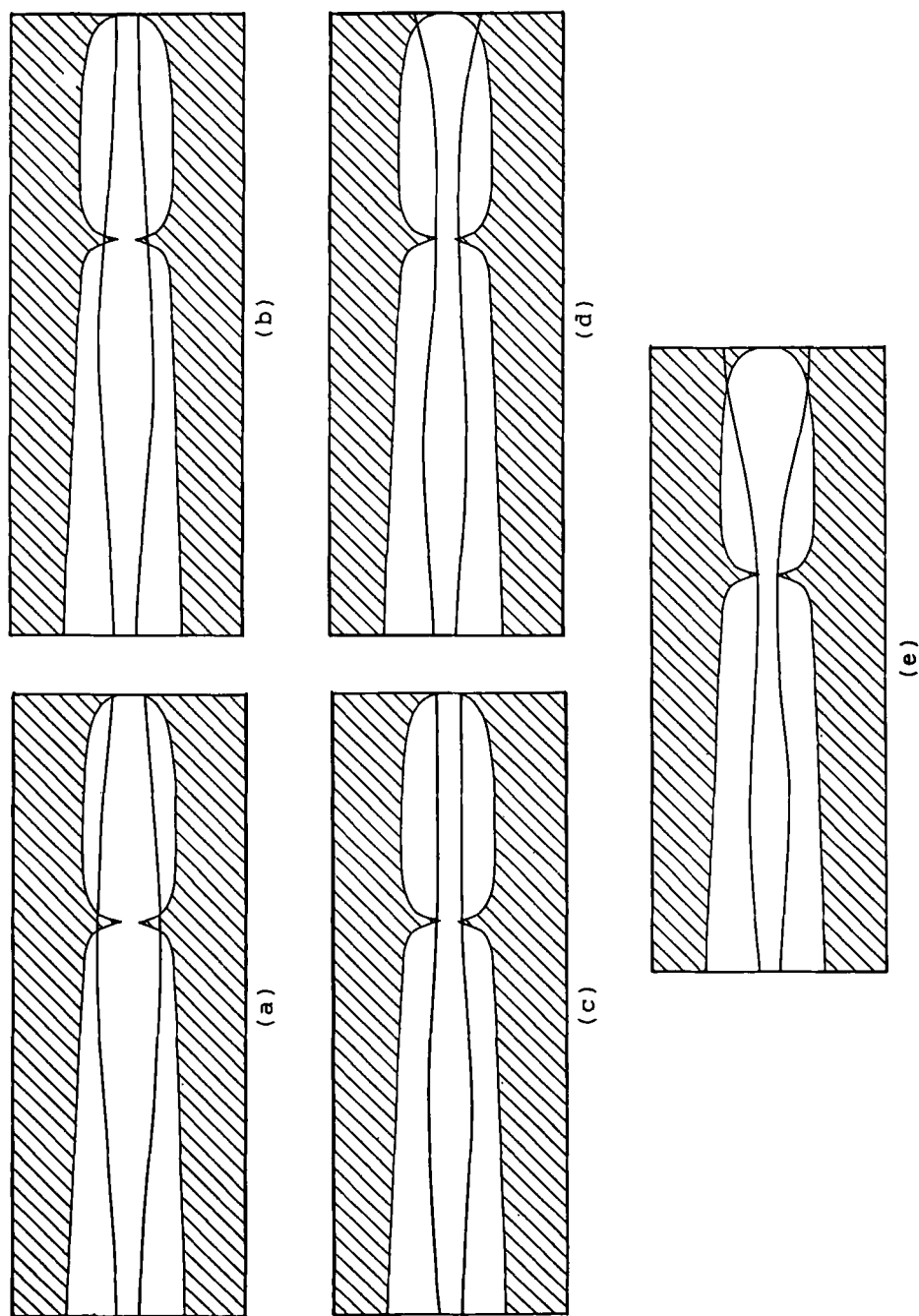


FIG. 2. Cross-sectional pictures of the channel model with the calculated envelope of the path of the ion in the center of each picture. In this and the Figs 3-5, the calculations are for the calcium ion. For Fig. 2, the d.c. magnetic field is $20 \mu\text{T}$, the a.c. peak magnetic field is $20 \mu\text{T}$, the a.c. peak magnetic field is $20 \mu\text{T}$ and the harmonic number is 1. The charge to mass resonance point for these conditions is 15-31 Hz. The frequencies shown are: (a) 9 Hz, (b) 12 Hz, (c) 15 Hz, (d) 18 Hz, (e) 21 Hz. Note the necessity of two restrictions, one inside the channel, one at the output end where the ion would enter the cytoplasm. Thus, at 18 and 21 Hz, the ion may pass under the inner restriction, but it will find that it has a considerable barrier at the output. The calculated path shown considers the outside edge of the ion. The path can be viewed as the envelope inside which the ion rolls as it follows a helical path through the channel.

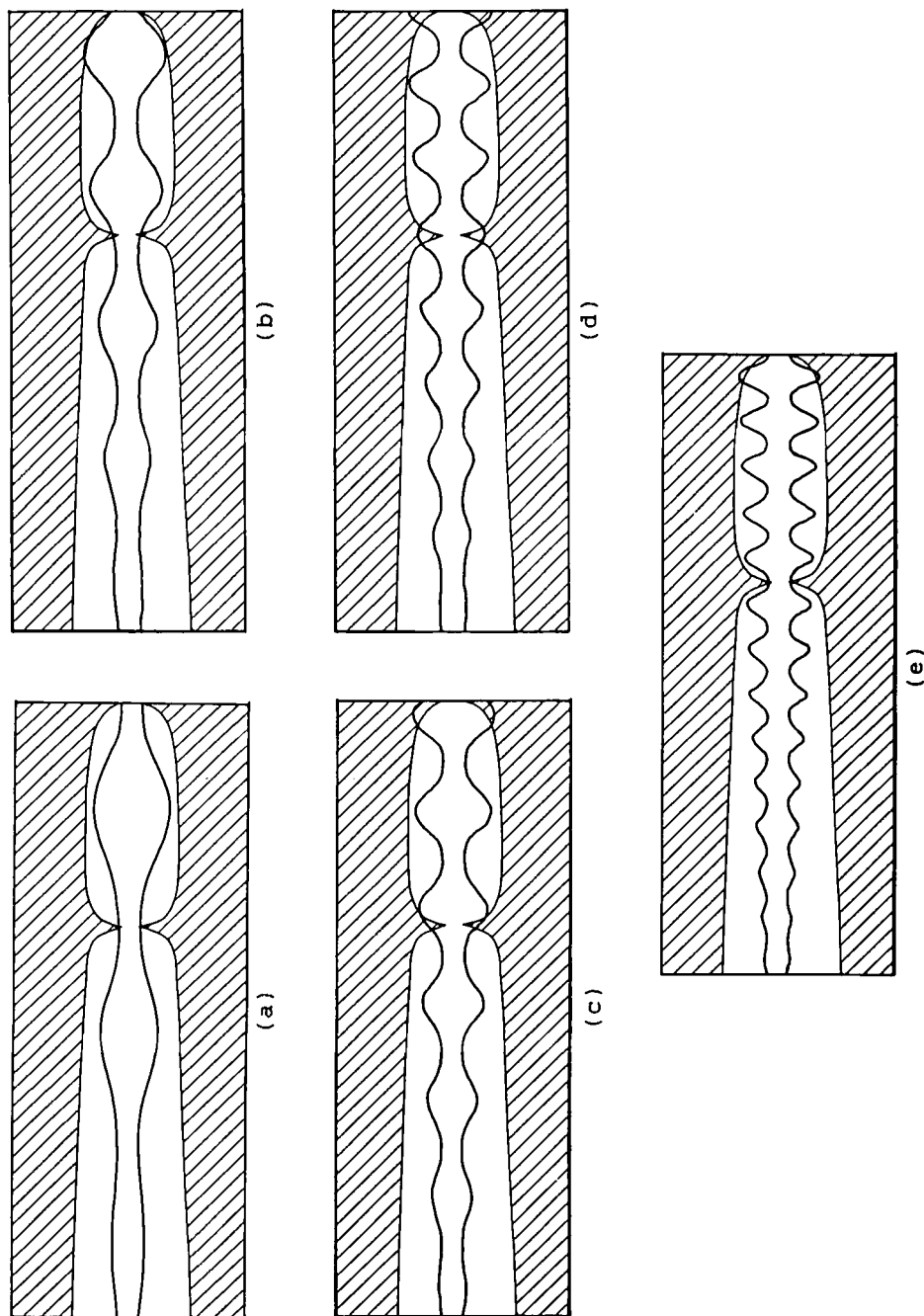


FIG. 3 (i). The frequency is set at 15 Hz, the other magnetic field conditions are set as in Fig. 2, but now the harmonic number is varied in each picture with: (a) $N=3$, (b) $N=5$, (c) $N=7$, (d) $N=9$, (e) $N=15$. For harmonic 1, see Fig. 2(c). Note that harmonics 1, 3, 5 and 15 fit the model channel relatively well, while ions at harmonics 7 and 9 will encounter the barriers both at the middle and at the end. For harmonics 5 and 15 the envelope of the ion's path is very close to the wall at the cytoplasmic end of the channel. In one of our systems (*Amphora coffeaeformis*), the data indicated that the response of the system was considerably lower at harmonic 5 as compared to harmonics 1 and 3. The response at 15 was fairly strong, however. (ii) The conditions are the same as for (i) but this set of figures shows the calculated ion path as the harmonic numbers are changed from 2 through 14 with (a) $N=2$, (b) $N=4$, (c) $N=6$, (d) $N=8$, (e) $N=10$, (f) $N=14$. Again, note that both barriers play a part in the blocking effect engendered by the channel shape. None of the even harmonics seem to fit the channel.

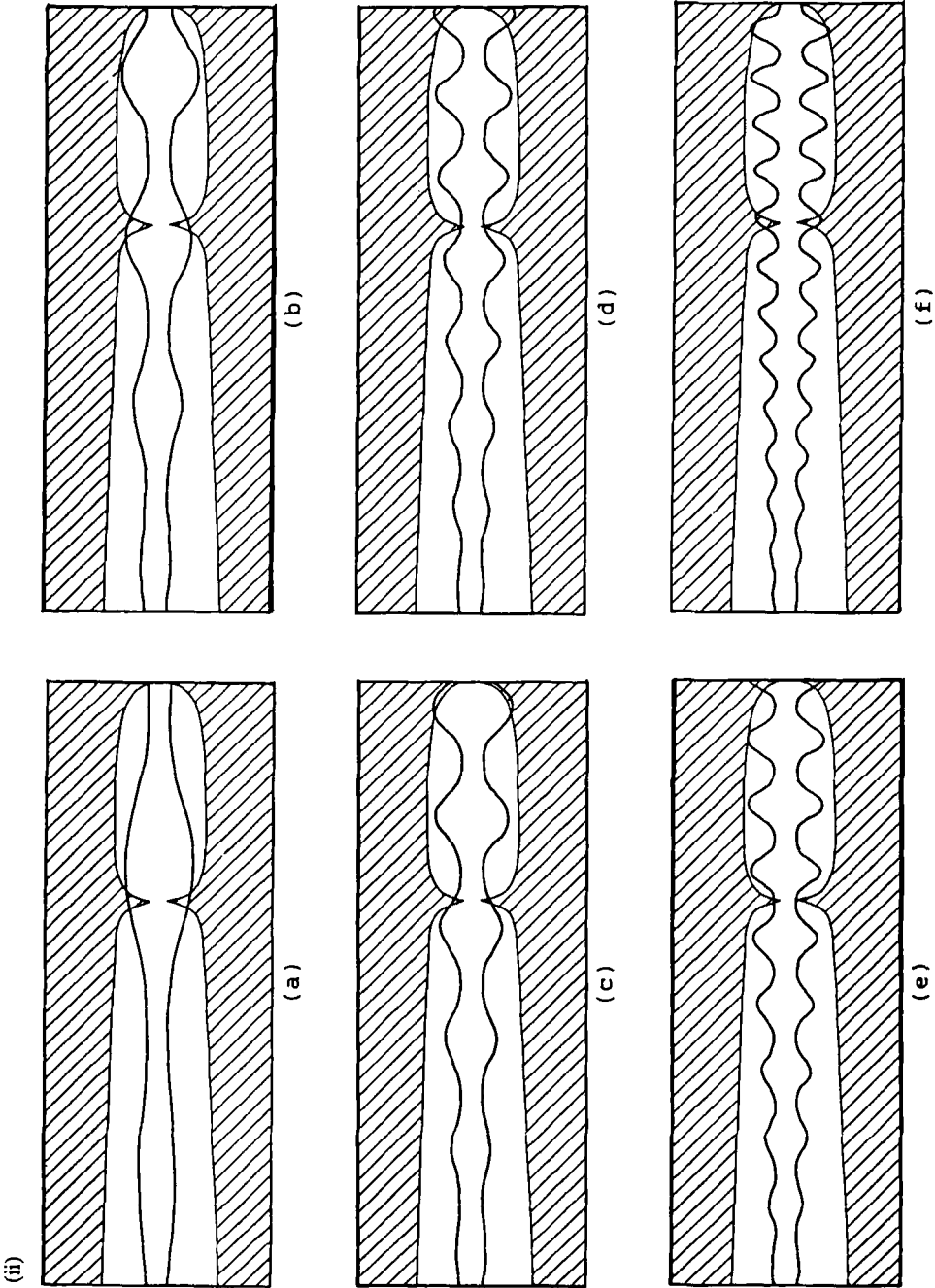


FIG. 3 continued.

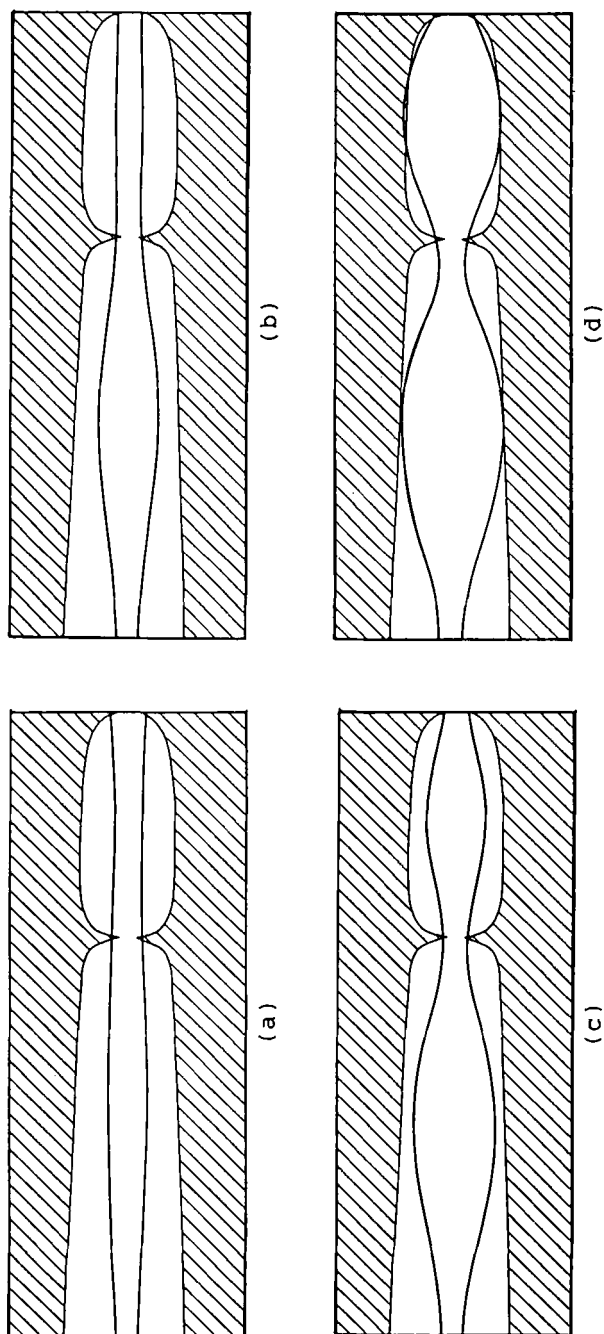


FIG. 4. This set of figures is included to indicate how the calculated ion path changes as the a.c. magnetic field peak amplitude is changed. The conditions are again for the calcium ion, the d.c. magnetic field is set at $20 \mu\text{T}$, the frequency set at 15 Hz , and the harmonic number set at 1 with the a.c. peak magnetic fields being $10 \mu\text{T}$, $30 \mu\text{T}$, $50 \mu\text{T}$, $70 \mu\text{T}$. It is observed that as the peak a.c. amplitude increases, the envelope also increases, eventually causing the ion to interfere with the channel walls. This effect is slightly different for different ions. For instance, if the magnesium ion charge to mass ratio is used at its resonance point, significant interference will occur when the a.c. magnetic field reaches about $45 \mu\text{T}$. There appears to be only one a.c. magnetic field "window" and it is rather broad.

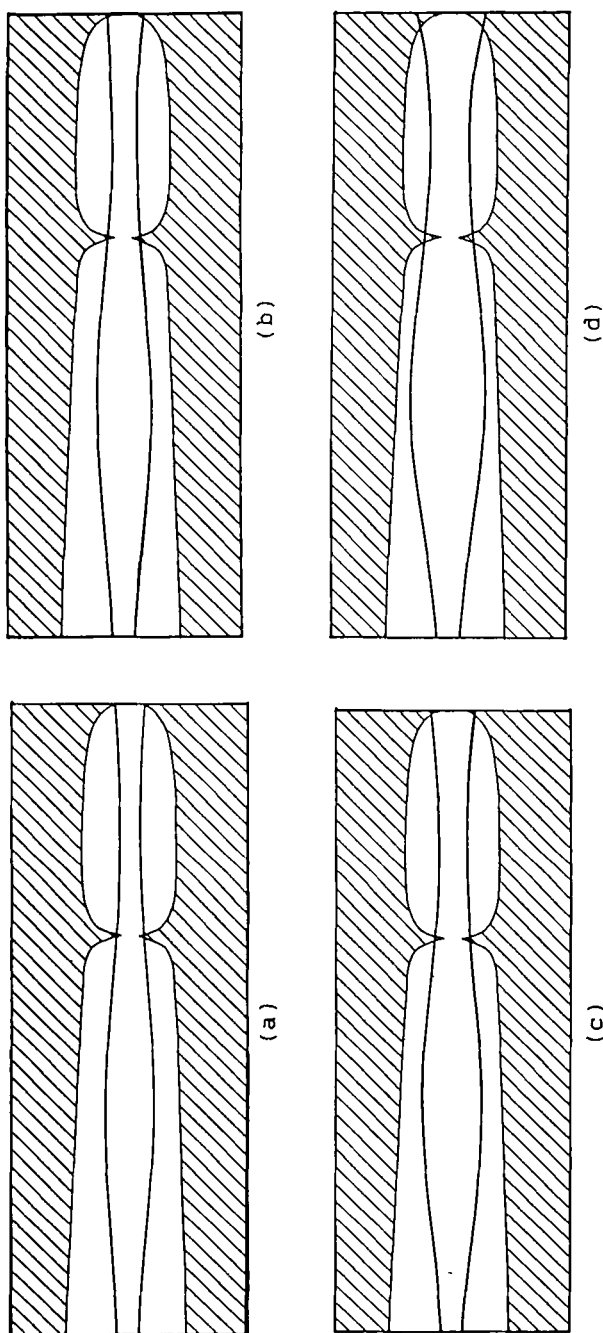


FIG. 5. Part of our data for *Amphiora coffeaeformis* indicated that one could change the d.c. magnetic field through a number of values and it was still possible to observe a peak response of the system at the qB/m point for a particular ion. We did find, however, that above a certain d.c. magnetic field, the response disappeared. This figure shows the calculated ion path for four different d.c. magnetic fields [(i) 15 μT , (ii) 25 μT , (iii) 30 μT and (iv) 40 μT]. The frequency for each picture is the resonance frequency for the calcium ion at that d.c. magnetic field, with the harmonic number set to 1 and the a.c. magnetic field set to 20 μT . These figures suggest that as the d.c. field is increased, the ion path will fit the channel at the resonance frequency up to a point, but finally a d.c. field will be reached where the fit is destroyed and the biosystem response would be expected to decrease.

odd harmonics 7, 9, 11 and 13. This grouping of allowed harmonics is consistent with previously reported experimental observations by this group (McLeod *et al.*, 1987).

Unlike other models (Chiabrera *et al.*, 1985; Durney *et al.*, 1988) and at least one experimental report (Blackman *et al.*, 1982), we find no indication from our results for the existence of *multiple* intensity windows, that is, multiple ranges of a.c. field intensities within which there are biological effects, but between which there are none. Instead, we find only one a.c. intensity range within which the magnetic field interaction permits ions to flow readily through the channel. At the same time, there is an upper cutoff on the intensity of the a.c. magnetic field ($B_{a.c.}$). This restriction of the ELF interaction to intensities below a certain level has significance for the general question of the dosimetry of ELF magnetic fields, reinforcing the argument (Blackman *et al.*, 1982) against the indiscriminate weighting of larger a.c. fields when attempting to extrapolate the epidemiological effect of weak ELF fields.

Further, the size of the ion may play an interesting role in this type of gate mechanism. All else remaining the same, it is to be expected that larger ions will have a corresponding sharper resonance (higher Q) as a function of the frequency of the a.c. field. In the one reported comparison, measured in terms of diatom motility (McLeod *et al.*, 1987), a much sharper frequency resonance was observed for K^+ -related effects as compared to Ca^{2+} -related effects.

We have suggested elsewhere (McLeod *et al.*, 1992) that the likelihood of an electromagnetic interaction with the cell increases as the cell is moved out of a state of equilibrium. Thus, it may be possible to define an equilibrium state from which the cell is perturbed, either by gross changes in the exogenous environment (e.g. temperature, ionizing radiation, pH) or by endogenous changes resulting from cell transformation or wound-healing demands. In this picture, these perturbations may serve to activate transient, low-conductance, highly selective channels that enable the cell to adjust transport through the membrane as it attempts to return to equilibrium. It is reasonable to further suggest that weak EM fields *per se* probably do not act to disturb cellular equilibrium, but that these fields may indeed interact with channels formed or activated in response to displacements from equilibrium caused by other factors. This argument appears to be consistent with the concept of a slowly-acting channel, akin to those found in the chromaffin cells in the adrenal gland, which provide a slow discharge of catecholamines for purposes of neural regulation following systemic stress. As pointed out above, the present work assumes a total time for channel passage of about 50 msec, a figure necessitated, in the final analysis, by the fact that we are specifically studying the interaction of ELF fields. This transit time is greatly in excess (by a factor of about 10^6) of the times associated with results from patch-clamp studies on excitable cells (Liboff, 1988, 1992). We are therefore led to the conclusion that one possible explanation for weak field, ELF interactions with cells is due to the existence of transient, ultra-slow membrane channels.

Finally, the presence of a single constriction within the lumen, as shown in Fig. 1, may be regarded as a first-order perturbation on the relatively smooth cylindrical lumens associated with simple polypeptide channels such as gramicidin A. It is interesting to note that arguments concerning facilitated permeation as a result of

magnetic resonance exposures are difficult to advance in great detail when one deals with perfectly cylindrical constraining walls. This is because any conceivable energy accretion as a result of resonance requires increasingly larger radii as the ion progresses through the channel, a fact that is inconsistent with the picture of a lumen having constant radius. Our present argument is not connected to the energy of the ion, but rather to the constraints imposed by the shape of the walls. The ions are not speeded up, but rather are gated through. Considering the great potential for complex protein-folding, one can imagine higher-order perturbations of channel symmetry, with two or more constricted regions along the lumen, leading to radically different electromagnetic gating outcomes (e.g. different allowed harmonics). In all such cases, however, the key role is played by the magnetic resonance frequency, a parameter dependent on the ion charge-to-mass ratio, which acts to ensure ion specificity.

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APPENDIX A

Definition of Terms

The vector fields are:

E = electric field intensity vector

$$= E_r a_r + E_\phi a_\phi + E_z a_z$$

F = reaction force vector inside the channel

$$= F_r a_r + F_\phi a_\phi + F_z a_z$$

B = applied magnetic field

$$= B_z a_z$$

$$B_z = B_0 + B_{ac} \sin(\omega t)$$

U = velocity vector

$$= U_r a_r + U_\phi a_\phi + U_z a_z$$

with: a_r, a_ϕ, a_z = unit vectors in the r, ϕ , and z directions

B_0 = magnitude of the d.c. magnetic field (Tesla)

$B_{a.c.}$ = peak amplitude of the a.c. magnetic field (Tesla)

q/m = charge to mass ratio of the ion (coulomb/kilogram)

$\omega = 2\pi f$ = applied radian frequency (Hertz)

$\omega_B = qB_z/m$

$$U_r = \frac{\partial r}{\partial t}$$

$$U_\phi = r \frac{\partial \phi}{\partial t}$$

$$U_z = \frac{\partial z}{\partial t}$$

τ = damping term (seconds)

a = ion radius (meters)

r_0 = channel radius (meters)

Development

The Lorentz force equation can be written in component form as:

$$\frac{\partial U_r}{\partial t} - \frac{\partial \phi}{\partial t} U_\phi = \left(\frac{q}{m}\right) E_r + \left(\frac{q}{m} B_z\right) (U_\phi) - \frac{U_r}{\tau} - \frac{F_r}{m} \quad (\text{A.1})$$

$$\frac{\partial U_\phi}{\partial t} + \frac{2}{r} \frac{\partial U_r}{\partial t} U_\phi - \frac{U_r}{r} U_\phi = \left(\frac{q}{m}\right) E_\phi - \left(\frac{q}{m} B_z\right) U_r - \frac{U_\phi}{\tau} - \frac{F_\phi}{m} \quad (\text{A.2})$$

$$\frac{\partial U_z}{\partial t} = \left(\frac{q}{m}\right) E_z - \frac{U_z}{\tau} - \frac{E_z}{m}. \quad (\text{A.3})$$

Assume E_r does not vary with time and also assume the reaction force F_r to be defined as

$$F_r = -m U_\phi \frac{\partial \phi}{\partial t} \quad (\text{A.4})$$

Suppose now that we take the time partial derivative of (A.1) and use (A.2) and (A.3) in the result. This leads to:

$$f(r, t) = \omega_B \left[\left(\frac{q}{m}\right) E_\phi - \frac{U_\phi}{\tau} - \frac{E_\phi}{m} \right] + \left(\frac{q}{m}\right) U_\phi \frac{\partial B_z}{\partial t} - G U_\phi \omega_B \quad (\text{A.5})$$

where

$$f(r, t) = \frac{\partial^2 U_r}{\partial t^2} + \frac{1}{\tau} \frac{\partial U_r}{\partial t} + \omega_B^2 U_r \quad (\text{A.6})$$

and

$$G = \frac{2}{r} \frac{\partial U_r}{\partial t} - \frac{1}{r} U_r \quad (\text{A.7})$$

Now, if we define F_ϕ as being

$$\frac{\omega_B}{rm} F_\phi = \left[\frac{\omega_B}{\tau} + G\omega_B \right] \frac{\partial \phi}{\partial t} \quad (\text{A.8})$$

or

$$F_\phi = \frac{m}{\tau} U_\phi + m \frac{\partial \phi}{\partial t} \left[2 \frac{\partial U_r}{\partial t} - U_r \right] \quad (\text{A.9})$$

that is, the F_ϕ reaction force is approximately equal to the collisional momentum term since the second two terms in (A.9) are probably quite small (assuming the velocity and its derivative are well behaved through the channel). This essentially suggests that the collision term is effectively reduced by a channel reaction force.

Then we may use (A.9) in (A.5), divide by r , and collect terms to yield

$$\frac{f(r, t)}{r} = g(\phi, r, t) \quad (\text{A.10})$$

where

$$g(\phi, r, t) = \frac{\partial \phi}{\partial t} \left[\left(\frac{q}{m} \right) \frac{\partial B_z}{\partial t} \right] + \left(\frac{q}{m} \right) \frac{\omega_B E_\phi}{r} \quad (\text{A.11})$$

and with $f(r, t)$ being defined in eqn (6).

If we use Faraday's law in the form

$$\oint \mathbf{E} \cdot d\mathbf{l} = - \frac{\partial}{\partial t} \int (\mathbf{B} \cdot d\mathbf{S}) \quad (\text{A.12})$$

then, since there is only a z directed component of the $\mathbf{B}$ field, $\mathbf{E}$ has the form

$$\mathbf{E}_\phi = - \left(\frac{r}{2} \right) \frac{\partial \mathbf{B}}{\partial t} - \mathbf{B} \frac{\partial r}{\partial t} \quad (\text{A.13})$$

We now use eqn (A.13) in (A.11), collect terms and then seek solutions to (A.10) which now has the form

$$\frac{f(r, t)}{r} + \left(\frac{\omega_B^2}{r} \right) U_r = \alpha = - \frac{q}{2m} \omega_B \frac{\partial \mathbf{B}}{\partial t} + \frac{q}{m} \frac{\partial \mathbf{B}}{\partial t} \frac{\partial \phi}{\partial t} \quad (\text{A.14})$$

where α is an arbitrary separation constant. Note that the left-hand (l.h.s.) side of (A.14) is a function of r and t only while the r.h.s. is a function of ϕ and t only. The most simple solutions will be for $\alpha=0$. Then we have

$$\frac{\partial^2 U_r}{\partial t^2} + \left(\frac{1}{\tau}\right) \frac{\partial U_r}{\partial t} + 2(\omega_B^2) U_r = 0 \quad (\text{A.15})$$

and

$$\frac{\partial \phi}{\partial t} = \frac{\omega_B}{2}. \quad (\text{A.16})$$

The solution to (A.16) is

$$\phi = \left[\frac{\omega_{B0}}{2} \right] t + \left[\frac{\omega_{Bac}}{2\omega} \right] \cos(\omega t) - \frac{\omega_{Bac}}{2\omega} \quad (\text{A.17})$$

(using $\phi=0$ at $t=0$ as an initial condition). Since the second and third terms in (A.17) both contain the inverse omega term, they will be small compared to the first term, assuming the applied frequency and the peak a.c. magnetic field are both relatively close to the resonance point. Thus, if the movement in the z direction is linear, eqn (A.17) describes a helical motion for the ion.

Equation (15) is straightforward to solve except that ω_B varies in time since the applied field has both an a.c. and a d.c. component. The equation was first handled as if ω_B was a constant, and the solution for (r) was checked in (A.15) to determine the extent of the error caused by this assumption. The resulting solution for (r) is then the same as eqn (A.1) in the Theory section:

$$r \approx \frac{(a-r_0)}{A} e^{-t/\tau} \left[\sinh\left(\frac{At}{2\tau}\right) + A \cosh\left(\frac{At}{2\tau}\right) \right] + (r_0 - a) \quad (\text{A.18})$$

where

$$A = \sqrt{1 - 8\omega_B^2 \tau^2} \quad (\text{A.19})$$

The boundary and initial conditions used were:

- (1) The initial and final velocities were taken as zero ($U_r=0$ at $t=0$ and as t approaches infinity).
- (2) The ion starts at the center of the mouth of the ion channel and the radial displacement cannot exceed the channel radius ($r=0$ at $t=0$ and r approaches (r_0-a) as t approaches infinity).

In order to make (A.18) a reasonable approximation, the factor (A) in eqn (A.18) must be kept in the range

$$0.9 \leq A \leq 1. \quad (\text{A.20})$$

The expression for z comes directly from (A.3) after assuming E_z and F_z do not vary with time. The result is

$$z \approx \frac{\tau q}{m} \left(E_z - \frac{F_z}{q} \right) t. \quad (\text{A.21})$$

Additional assumptions used to obtain z were that the ion starts from rest, and that the factor τ remains less than about 1 msec.

An alternative way of defining F_ϕ would be to suggest in eqn (A.5) that the entire r.h.s. is zero. That is, define F_ϕ as the constraining force

$$F_\phi = m \left[\frac{q}{m} E_\phi - U_\phi \left(\frac{1}{\tau} + G - \frac{1}{B} \frac{\partial B}{\partial t} \right) \right]. \quad (\text{A.22})$$

This would allow us to go directly to an equation like (A.15) except there would be no multiple of 2 in the third term on the left. Equation (A.19) would be modified in that the 8 would become 4 under the radical. The effect on the solutions for the motion of the ion is rather small, since the change is proportional to the square root of 2. If one follows this assumption, it is not possible to directly solve for the ϕ variable as was done in (A.17). It could be argued that (A.22) is a more physically realistic approach, but note that both (A.9) and (A.22) suggest that the ion proceeds down the channel with reduced collisions.